

Scheduling at the Swarthmore College Writing Center

Operations Research Case Study Project Final Report

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Writing Associates
SWARTHMORE COLLEGE



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Abstract

The aim of this project was to employ an operations research approach to optimize the scheduling of Writing Associates (WAs) in the Swarthmore College Writing Center. Previously, this scheduling had been completed manually at the beginning of each semester according to loosely defined priorities. The utilization of operations research techniques allows for the creation of a schedule that optimally conforms to a number of goals and constraints. For this project, an algebraic formulation and an AMPL model have been completed based on guidance from the client. The AMPL model was successfully solved with the CPLEX solver for a test data set of eleven WAs. Strategies of data collection and results formatting have been suggested to aid in the implementation of this operations research approach.

Introduction

The Swarthmore College Writing Center is a drop-in center where students submit papers for conferences with Writing Associates. Approximately twenty-five trained students known as Writing Associates work weekly shifts in the Writing Center throughout the semester. As shown in Figure 1 below, there are three overlapping shifts in each of six sessions (Sunday afternoon, Sunday evening, and Monday-Thursday evenings) for a total of eighteen weekly shifts. One or more WAs may work a given shift. WAs typically work one two-hour shift per week, but some opt to work more.

Sessions											
Sunday Afternoon		Sunday Evening		Monday Evening		Tuesday Evening		Wednesday Evening		Thursday Evening	
1-3	Shift	7-9	Shift	7-9	Shift	7-9	Shift	7-9	Shift	7-9	Shift
2-4	Shift	8-10	Shift	8-10	Shift	8-10	Shift	8-10	Shift	8-10	Shift
3-5	Shift	9-11	Shift	9-11	Shift	9-11	Shift	9-11	Shift	9-11	Shift

Figure 1 - Shifts and Sessions in a Week

In the current system, students who will be working in the Writing Center fill out an online survey listing times during which they are unavailable, their specialties (e.g. foreign languages or technical papers), and their shift preferences. Shift preferences are indicated through an open-ended text response. The Writing Associate Scheduling Coordinator then downloads a compilation of these responses and manually creates a schedule.

The goals of the completed schedule are: to accommodate the preferences expressed by the Writing Associates, to ensure that the workforce on each shift is proportional to the historical level of Writing Center usage during that shift (e.g. Sunday evening shifts tend to have more traffic than Wednesday evening shifts), and to distribute specialist Writing Associates so that there is even coverage throughout the week (e.g. Writing Associates who can work with Spanish language papers should not all be grouped in one night). These goals are subject to the

constraints of the Writing Associates' schedules; there are certain shifts for which they are absolutely unavailable.

The Coordinator Team of the Writing Associates Program consists of three faculty/staff members and three students. The Director of the Program and the primary contact for this project is Professor Jill Gladstein. I am serving as the 2008-2009 Scheduling Coordinator for the Program. The possibility of implementing an operations research approach to scheduling was raised in a conversation in the spring of 2008 with the 2007-2008 coordinators.

This project has been developed in collaboration with the Writing Center Coordinator Team. As an operations research analyst, I have incorporated their guidance and feedback into a recommended series of tasks to implement an optimization approach. This report details the four stages of this case study project: problem identification, formulation, solution, and implementation.

Problem Identification and Project Goals

The current system creates a large amount of work for the Scheduling Coordinator. In the past, Scheduling Coordinators have created schedules by rearranging WAs until all of their availabilities were satisfied and a good combination of preferences seemed to have been reached. In the Fall of 2008, WAs were placed into a Microsoft Outlook Calendar and moved around until a feasible (but possibly sub-optimal) schedule was completed (See Figure 2).

This manual approach can be tedious, since the addition of one additional WA can lead to the rearrangement of the entire schedule. It can also be a tedious task to sort through each of the WAs' preferences. Responses to the open-ended text response prompt "please describe any preferences for when your shift will be scheduled" vary widely and are often ambiguous. Responses to this prompt in Fall 2007 ranged from "1. Tuesday, 2. Wednesday, 3. Monday... I'd prefer to be at the earlier end of the shifts on all these days. Thanks!" to "in the evening" to blank responses. Deciding how to interpret these qualitative responses involves a tremendous amount of guesswork.

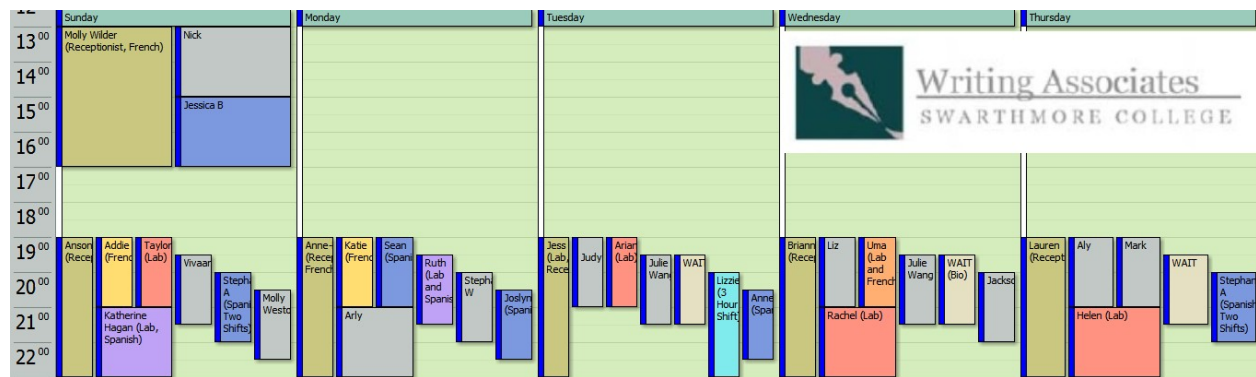


Figure 2 - Schedule for Fall 2008 Created Manually with Microsoft Outlook

Furthermore, the schedule created may not optimize the distribution of Writing Associates, both in terms of demand and specialties. The sheer number of scheduling combinations and

constraints makes exhaustive enumeration infeasible. Relying on manual schedule creation raises the possibility that an ideal schedule will not be implemented because it is simply not considered. This system also raises questions of fairness. The Scheduling Coordinator can conceivably prioritize the preferences of his or her acquaintances when assigning WAs to shifts.

An ideal operations research formulation will optimize the distribution of the Writing Associates and reduce concerns about fairness. It will also reduce the time spent creating the schedule and lead to a more precise, quantitative process.

The optimal solution to this formulation may not be the best solution to actually implement. Factors outside of the formulation should be taken into account in revising what operations research defines as the optimal solution. For example, WAs that the Director knows to be slow workers should not be placed in closing shifts, and two WAs who have personal issues with one another should not be placed together in the same session.

My implementation thus aims to give the Coordinator Team an optimal, feasible starting schedule that can be adapted to concerns outside of the realm of quantitative formulation. Based on input from the Program Director and the Coordinator Team, as well as my own experience, a successful implementation:

- Does not create a burden for WAs in entering their own scheduling data
- Is sufficiently documented for future Scheduling Coordinators
- Is customizable and generalizable for future Writing Center needs
- Outputs an easily understandable schedule

Formulation Goals

This formulation employs a goal-programming technique. Three goals have been identified by the client:

Goal 1: Shift Targets

Based on historical usage trends, there is a different numbers of WAs desired to work different shifts. For example, early on a Sunday evening tends to have more traffic than late on a Thursday evening. Thus, the target for the number of WAs required at 7:00 on Sunday might be three, while the target number of WAs required at 9:00 on Thursday might be one.

Goal 2: Specialties

WAs with specialties should be evenly distributed throughout the week. For example, this goal penalizes solutions with shifts that deviate from having an even distribution of WAs proficient in Spanish.

Goal 3: WA Preferences

WAs will prefer to work some shifts over others based on their personal schedules. Each WA may rank each shift as highly preferable ($P_{i,j}=9$), preferable ($P_{i,j}=7$), possible ($P_{i,j}=4$), not

preferable ($P_{i,j}=2$), or infeasible ($P_{i,j}=-1$). This goal penalizes solutions in which WAs do not work shifts they highly prefer.

Weighting

The client agreed that Goal 1 was the most important goal. It was arbitrarily assigned a weight of four, while the other goals received weights of one.

Mathematical Formulation

$i=1 \dots m$ represents individual WAs

$j=1 \dots n$ represents shifts in the Writing Center (e.g. Sunday starting at 7:00 or Tuesday starting at 9:00).

$k=1 \dots p$ represents specialties (currently Spanish, French, and Biology, so $p=3$).

t_j is a parameter indicating the target number of WAs to staff shift j .

w_i is a parameter indicating the number of shifts WA i wants to work.

$P_{i,j}$ is a parameter indicating WA i 's preference for working shift j .

$x_{i,j}$ is a binary decision variable that equals 1 if WA i is working on shift j .

$s_{i,k}$ is a binary decision variable that equals 1 if WA i is a specialist in subject k .

Goal 1: Shift Targets

$$\sum_{i=1}^m x_{i,j} \geq t_j \text{ for } j=1 \dots n$$

$$\text{so } y_{1j} = t_j - \sum_{i=1}^m x_{i,j}$$

Goal 2: Specialties

$$\sum_{i=1}^m x_{i,j} \geq \frac{1}{k} \sum_{i=1}^m s_{i,k} \text{ for } j=1 \dots n \text{ and } k=1 \dots p$$

$$\text{so } y_{2j,k} = \frac{1}{k} \sum_{i=1}^m s_{i,k} - \sum_{i=1}^m s_{i,k} x_{i,j}$$

Goal 3: WA Preferences

$$\sum_{i=1}^m \sum_{j=1}^n P_{i,j} x_{i,j} \geq 9m$$

$$\text{so } y_3 = \sum_{i=1}^m \sum_{j=1}^n (9 - P_{i,j} x_{i,j})$$

Formulation

$$\text{Objective: Minimize } z = 4 \sum_{j=1}^n |y_{1j}| + \sum_{j=1}^n \sum_{k=1}^p |y_{2j,k}| + y_3$$

The absolute value functions are required for goals 1 and two because there is a one to one correspondence between positive deviations and negative deviations. For example, with a fixed number of WAs and the total number of WAs equal to the total of shift targets, one shift exceeding its target by one WA will require one shift missing its target by one WA. Thus, the absolute value of the deviations from goals 1 and 2 is required for these goals to be effective.

Subject to:

$$\text{Constraint 1: Workload } \sum_{j=1}^n x_{i,j} = w_i \text{ for } i=1 \dots m$$

WAs are required to work at least one shift in the Writing Center, but cannot be forced to work more than one shift if they do not desire to.

$$\text{Constraint 2: Availability } P_{i,j} x_{i,j} \geq 0$$

When WAs indicate their preferences, absolute unavailability is indicated by $P=-1$. This constraint ensures that no solution is feasible if any one WA is absolutely unavailable for his or her shift.

Test Data Set

WAs are only scheduled at the beginning of each semester, so testing of this model with actual data was impossible. Accordingly, a test data set of eleven WAs was compiled through entry into Survey Monkey's survey instrument. Mock WA names followed the pattern WaAlpha, WaBravo, WaCharlie, etc. A sample of the survey output is given in Appendix A. These data were then converted into the spreadsheets given in Appendix B. Eleven was the maximum allowable number of WAs for use in AMPL Student Edition for Taha Textbook, which limits the number of variables to 300. Access to a commercially licensed installation with no limits on variables was obtained, but this installation was incompatible with the ODBC table handler required by this formulation's data file.

Since the number of WAs was limited, large workloads (≤ 4 shifts per week) were assigned to some WAs. While it is unrealistic to assume that real WAs would be willing to work so many shifts, this test data set was a fairly accurate representation of actual scheduling constraints given the situation.

AMPL Model Code

The AMPL model used in this formulation is included with illustrative comments bolded below. The run and data files, as well as AMPL's solution, are included in Appendix C.

Model File

```
#AMPL file final.mod

set WAS ordered by ASCII;
set SHIFTS ordered;
set SPECIALTIES;
set PREFERENCES={WAS, SHIFTS};
set SPECIALISTS={WAS, SPECIALTIES};

param pref {PREFERENCES};
param target {SHIFTS};
param spec {SPECIALISTS};
param workload {WAS};

var work {PREFERENCES} binary;
    #Work will be a binary variable equal to 1 if WA i works on shift j
var goal1 {SHIFTS};
    #goal1 represents the shift target goal and is defined in a constraint below. It is required to supply a simple variable, instead of an expression, for the argument of the piecewise linear function in the objective function
var goal2 {SHIFTS, SPECIALTIES};
    #goal2 represents the specialties goal and is defined in a constraint below. It is required to supply a simple variable, instead of an expression, for the argument of the piecewise linear function in the objective function

minimize z:

4*sum{j in SHIFTS}<<0; -1,1>>goal1[j]+
sum{j in SHIFTS, k in SPECIALTIES}<<0; -1,1>>goal2[j,k]+
sum{(i,j) in PREFERENCES}(9-pref[i,j]*work[i,j]);
    #Because the CPLEX 9.1.3 solver needed to process the binary variables does not work with the absolute value function, the above explicit piecewise linearization is necessary. MINOS, another available solver, is able to handle absolute value functions, but not binary variables.

subject to staffing {j in SHIFTS}:
goal1[j]=target[j]-sum{i in WAS}work[i,j];
```



```

subject to spe {j in SHIFTS, k in SPECIALTIES}:
goal2[j,k]=sum{i in WAS}spec[i,k]/card(SHIFTS)-sum{I in
WAS}spec[i,k]*work[i,j];
#card(SHIFTS) returns the total number of shifts. So sum{i in
WAS}spec[i,k]/card(SHIFTS) represents the number of WAs with
specialty k that would work on each shift if they were distributed
evenly.

```

#The following two constraints are hard constraints:

```

subject to limit {i in WAS}:
sum {j in SHIFTS} work[i,j] = workload[i];

subject to availability {i in WAS, j in SHIFTS}:
pref[i,j]*work[i,j] >= 0;

```

Solution and Implementation

Implementation relies on four steps: data collection, data preparation, processing, and results formatting.

Data Collection

Preferences and specialties need to be gathered from each of the WAs who will be working in the Writing Center. Different means of data collection were explored. Both Google Spreadsheets and SurveyMonkey convert responses gathered from users in a browser-based interface into spreadsheets. These spreadsheets can then be formatted into tables readable by AMPL.

Survey Monkey was selected for its relatively better customizability and for end-user familiarity, as past Scheduling Coordinators have used it to gather responses. Past surveys, the links to which have been emailed to WAs each semester, have been as long as three separate pages. The survey created for this implementation was reduced to one page to reduce the time required of WAs (See Figure 3). To reduce the variability and ambiguity of shift preference response, it is now mandatory to indicate one preference for all shifts.

Data Preparation

Data is downloaded from Survey Monkey as an .xls file (See Appendix A). Through a series of transpose and cut-and-paste operations, the data in this file are reformatted into four named ranges (See Appendix B) readable by AMPL's ODBC table handler. This is a process that can be more fully automated by the use of Excel Macros.

Processing

The file final.run is run in AMPL. For the test data set of eighteen WAs, the CPLEX solver employed by AMPL requires 354 MIP simplex iterations and 35 branch-and-bound nodes to reach an optimal solution (See Appendix C). The optimal shift assignments are then exported to an Excel spreadsheet as three columns (See Appendix D).

Writing Center Scheduling

Name (First and Last)

Class Year
 2011 2010 2009

Specialties
 Spanish French Bio

Indicate a preference for each shift

	I would love this shift	I would like this shift	I could take this shift	I would strongly prefer not to take this shift	I am absolutely unavailable for this shift
Sunday 1-3	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Sunday 2-4	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Sunday 3-5	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Sunday 7-9	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Sunday 8-10	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Sunday 9-11	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Monday 7-9	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
Monday 8-10	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
Monday 9-11	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
Tuesday 7-9	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
Tuesday 8-10	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
Tuesday 9-11	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
Wednesday 7-9	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
Wednesday 8-10	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
Wednesday 9-11	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
Thursday 7-9	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
Thursday 8-10	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
Thursday 9-11	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>

Please add any additional requests here.

Figure 3 - Sample responses in web-based survey instrument

Results Formatting

In one step, the raw output spreadsheet can be converted to a more easily readable Pivot Table (See Appendix E). From this pivot table, it is easy to read off shift assignments and type them into a shift-by-shift schedule:

		Work Start	Work Duration
Sunday		13:00	10:00
	Receptionist	13:00	04:00
	WaEcho	13:00	02:00
	WaAlpha	14:00	02:00
	WaHotel	15:00	02:00
	Receptionist	19:00	04:00
	WaBravo	19:00	02:00
	WaJuliet	19:00	02:00
	WaEcho	20:00	02:00
	WaHotel	20:00	02:00
	WaDelta	21:00	02:00
Monday		13:00	10:00
	Receptionist	19:00	04:00
	WaEcho	19:00	02:00
	WaGolf	19:00	02:00
	WaJuliet	20:00	02:00
	WaKilo	21:00	02:00
Tuesday		13:00	10:00
	Receptionist	19:00	04:00
	WaFoxtrot	19:00	02:00
	WaGolf	20:00	02:00
	WaLima	21:00	02:00
Wednesday		13:00	10:00
	Receptionist	19:00	04:00
	WaLima	19:00	02:00
	WaEcho	19:00	02:00
	WaDelta	20:00	03:00
Thursday		13:00	10:00
	Receptionist	19:00	04:00
	WaDelta	19:00	04:00
	WaFoxtrot	19:00	02:00
	WaCharlie	20:00	02:00

The format of the above table is read directly into an Excel custom floating bars chart. This chart indicates WAs' specialties with color codes, enabling a quick and efficient visual assessment of the distribution of WAs and specialties across the week.

Writing Center Schedule

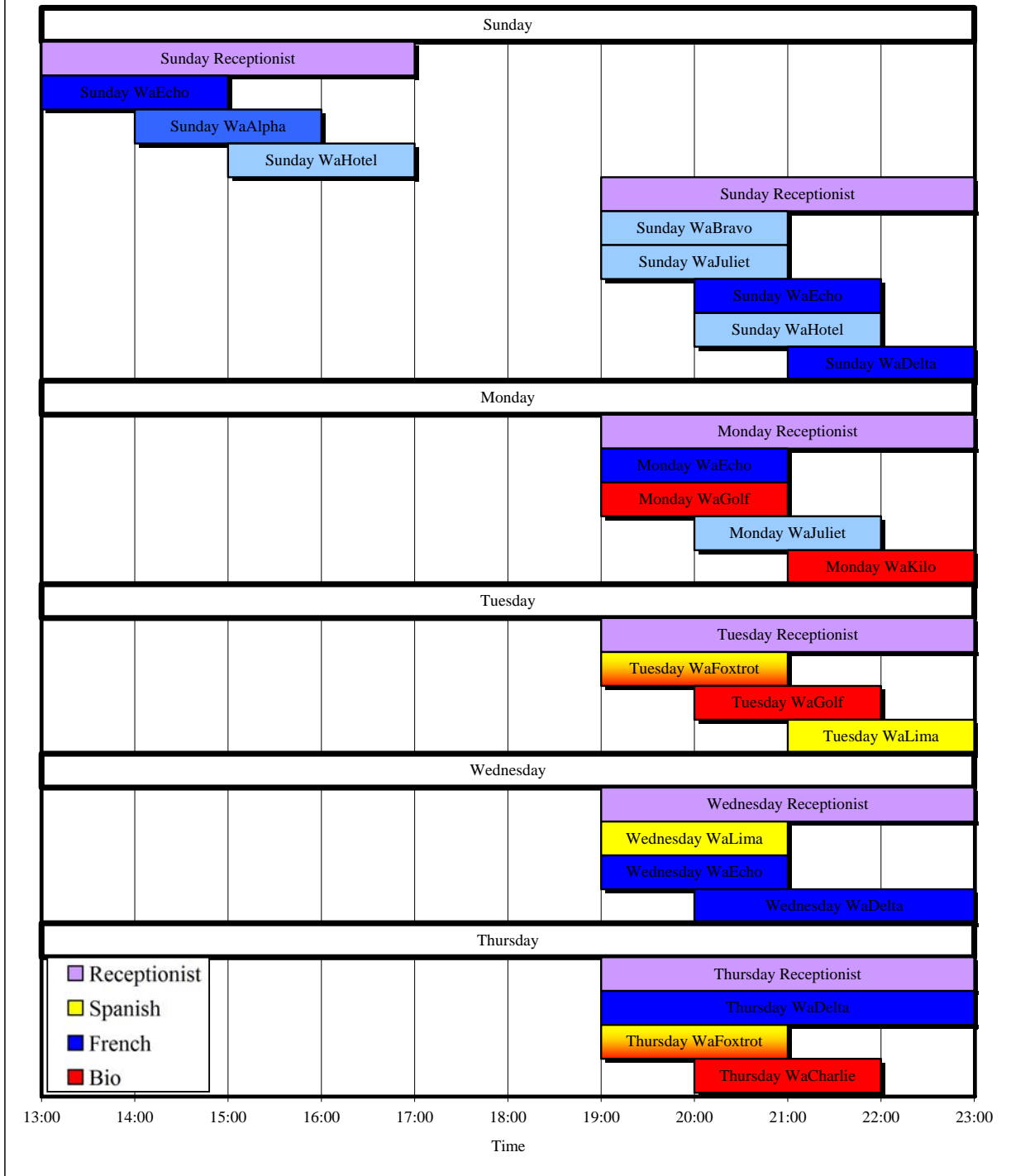


Figure 4 - Finalized Schedule as an Excel Custom Floating Bars Chart

Client Feedback

Professor Gladstein was informed of the status of this project throughout its evolution. She provided guidance especially regarding the formulation of the operations research model. A final presentation of the applicability and methods of this operations research approach was made to the entire Writing Associate Program Coordinator Team on December 1, 2008. Feedback was generally positive. Professor Gladstein predicts that this optimization approach will save Scheduling Coordinators significant amounts of time.

Some suggestions about the implementation given at this meeting changed the formulation. For example, the Coordinators argued that it was too burdensome and repetitive for WAs to fill out one page indicating absolute unavailability, then another page indicating preferences for all shifts. Incorporating this suggestion required a departure from the original plans for the unavailability constraint.

Conclusions and Future Research

This project was successful both in its end product and in the experience I gained with operations research. I considered challenges such as goal programming, non-integer programming, and AMPL formulations all within the context of an identified problem and a practical plan for implementation. Throughout this project's development, concerns about real-world implementation constantly required the revision of my formulation and subsequent solutions.

Further steps should be taken to ensure completely successful implementation. Additional consultation with the client is necessary to determine the appropriateness of weighting in the formulation's objective function. For example, the current weighting does not result in a very even distribution of specialist WAs, and the client may prefer to prioritize this goal at the expense of meeting WAs' requests for most preferable shifts. Optimal weighting will have to be determined by trial-and-error investigation.

The workflow implemented for this case study is infeasible with the full number of WAs because it relies on the ODBC table handler. It is thus imperative to develop an efficient workflow that formats the input and output data for AMPL as tab separated text (.tab) files rather than one .xls spreadsheet with multiple named ranges. A possible approach is the use of Excel Macros to automatically convert the named ranges of parameters into separate .tab files. Further documentation of all four stages of implementation, data collection, data preparation, processing, and results formatting, is required for use by future Scheduling Coordinators.

Acknowledgements

Writing Associates Program Director Jill Gladstein has been helpful and supportive throughout this project. I am also thankful for the encouragement of the past and present Coordinators Lucy McNamara, Brianna Gallagher, Molly Wilder, and especially 2007-2008 Scheduling Coordinator Lisa Benson. Finally, this project would have been impossible without Professor McGarity's instruction and guidance in modeling, linear programming, and AMPL.

Appendix A – Sample Survey Data Returned by SurveyMonkey

(Split into multiple rows to fit page)

RespondentID	StartDate	EndDate	IP Address	Email Address	First Name	LastName	Custom Data
698362695	12/03/2008	12/03/2008	130.58.231.149				

Name (First and Last) Open-Ended Response WaAlpha	Class Year Response	Specialties Spanish	French	Bio	Indicate a preference for each shift Sunday 1-3	Sunday 2-4	9
	2				3		9

Sunday 3-5	Sunday 7-9	Sunday 8-10	Sunday 9-11	Monday 7-9
9	4	4	4	-1

Monday 8-10	Monday 9-11	Tuesday 7-9	Tuesday 8-10	Tuesday 9-11
-1	-1	2	2	2

Tuesday 7-9	Wednesday 8-10	Wednesday 9-11	Thursday 7-9	Thursday 8-10
2	2	2	2	2

Thursday 9-11	Please add any additional requests here. Open-Ended Response
2	

Appendix B – Test Input Data

Preferences Table

WAS	SHIFTS	pref	WaEcho	Tuesday7	-1	WaAlpha	Sunday2	9
WaGolf	Sunday1	-1	WaEcho	Tuesday8	-1	WaAlpha	Sunday3	9
WaGolf	Sunday2	-1	WaEcho	Tuesday9	-1	WaAlpha	Sunday7	4
WaGolf	Sunday3	9	WaEcho	Wednesday7	9	WaAlpha	Sunday8	4
WaGolf	Sunday7	2	WaEcho	Wednesday8	9	WaAlpha	Sunday9	4
WaGolf	Sunday8	2	WaEcho	Wednesday9	9	WaAlpha	Monday7	-1
WaGolf	Sunday9	2	WaEcho	Thursday7	4	WaAlpha	Monday8	-1
WaGolf	Monday7	9	WaEcho	Thursday8	4	WaAlpha	Monday9	-1
WaGolf	Monday8	4	WaEcho	Thursday9	4	WaAlpha	Tuesday7	2
WaGolf	Monday9	4	WaDelta	Sunday1	9	WaAlpha	Tuesday8	2
WaGolf	Tuesday7	4	WaDelta	Sunday2	9	WaAlpha	Tuesday9	2
WaGolf	Tuesday8	9	WaDelta	Sunday3	9	WaAlpha	Wednesday7	2
WaGolf	Tuesday9	7	WaDelta	Sunday7	9	WaAlpha	Wednesday8	2
WaGolf	Wednesday7	7	WaDelta	Sunday8	9	WaAlpha	Wednesday9	2
WaGolf	Wednesday8	7	WaDelta	Sunday9	9	WaAlpha	Thursday7	2
WaGolf	Wednesday9	7	WaDelta	Monday7	7	WaAlpha	Thursday8	2
WaGolf	Thursday7	-1	WaDelta	Monday8	7	WaAlpha	Thursday9	2
WaGolf	Thursday8	9	WaDelta	Monday9	7	WaHotel	Sunday1	9
WaGolf	Thursday9	-1	WaDelta	Tuesday7	2	WaHotel	Sunday2	9
WaFoxtrot	Sunday1	2	WaDelta	Tuesday8	9	WaHotel	Sunday3	9
WaFoxtrot	Sunday2	2	WaDelta	Tuesday9	-1	WaHotel	Sunday7	9
WaFoxtrot	Sunday3	2	WaDelta	Wednesday7	-1	WaHotel	Sunday8	9
WaFoxtrot	Sunday7	2	WaDelta	Wednesday8	9	WaHotel	Sunday9	9
WaFoxtrot	Sunday8	2	WaDelta	Wednesday9	9	WaHotel	Monday7	9
WaFoxtrot	Sunday9	4	WaDelta	Thursday7	9	WaHotel	Monday8	2
WaFoxtrot	Monday7	4	WaDelta	Thursday8	9	WaHotel	Monday9	2
WaFoxtrot	Monday8	4	WaDelta	Thursday9	9	WaHotel	Tuesday7	2
WaFoxtrot	Monday9	4	WaBravo	Sunday1	-1	WaHotel	Tuesday8	2
WaFoxtrot	Tuesday7	7	WaBravo	Sunday2	-1	WaHotel	Tuesday9	2
WaFoxtrot	Tuesday8	7	WaBravo	Sunday3	-1	WaHotel	Wednesday7	2
WaFoxtrot	Tuesday9	7	WaBravo	Sunday7	9	WaHotel	Wednesday8	2
WaFoxtrot	Wednesday7	-1	WaBravo	Sunday8	9	WaHotel	Wednesday9	-1
WaFoxtrot	Wednesday8	-1	WaBravo	Sunday9	9	WaHotel	Thursday7	-1
WaFoxtrot	Wednesday9	-1	WaBravo	Monday7	7	WaHotel	Thursday8	-1
WaFoxtrot	Thursday7	9	WaBravo	Monday8	7	WaHotel	Thursday9	-1
WaFoxtrot	Thursday8	9	WaBravo	Monday9	7	WaJuliet	Sunday1	-1
WaFoxtrot	Thursday9	9	WaBravo	Tuesday7	2	WaJuliet	Sunday2	-1
WaEcho	Sunday1	9	WaBravo	Tuesday8	2	WaJuliet	Sunday3	-1
WaEcho	Sunday2	9	WaBravo	Tuesday9	2	WaJuliet	Sunday7	9
WaEcho	Sunday3	9	WaBravo	Wednesday7	2	WaJuliet	Sunday8	2
WaEcho	Sunday7	9	WaBravo	Wednesday8	2	WaJuliet	Sunday9	2
WaEcho	Sunday8	9	WaBravo	Wednesday9	2	WaJuliet	Monday7	2
WaEcho	Sunday9	9	WaBravo	Thursday7	4	WaJuliet	Monday8	9
WaEcho	Monday7	9	WaBravo	Thursday8	9	WaJuliet	Monday9	-1
WaEcho	Monday8	2	WaBravo	Thursday9	4	WaJuliet	Tuesday7	-1
WaEcho	Monday9	2	WaAlpha	Sunday1	9	WaJuliet	Tuesday8	7

WaJuliet	Tuesday9	7	WaLima	Monday8	4
WaJuliet	Wednesday7	4	WaLima	Monday9	2
WaJuliet	Wednesday8	4	WaLima	Tuesday7	2
WaJuliet	Wednesday9	4	WaLima	Tuesday8	2
WaJuliet	Thursday7	4	WaLima	Tuesday9	9
WaJuliet	Thursday8	4	WaLima	Wednesday7	9
WaJuliet	Thursday9	4	WaLima	Wednesday8	9
WaCharlie	Sunday1	2	WaLima	Wednesday9	-1
WaCharlie	Sunday2	2	WaLima	Thursday7	-1
WaCharlie	Sunday3	2	WaLima	Thursday8	9
WaCharlie	Sunday7	2	WaLima	Thursday9	-1
WaCharlie	Sunday8	2			
WaCharlie	Sunday9	2			
WaCharlie	Monday7	2			
WaCharlie	Monday8	2			
WaCharlie	Monday9	-1			
WaCharlie	Tuesday7	-1			
WaCharlie	Tuesday8	-1			
WaCharlie	Tuesday9	2			
WaCharlie	Wednesday7	2			
WaCharlie	Wednesday8	2			
WaCharlie	Wednesday9	2			
WaCharlie	Thursday7	2			
WaCharlie	Thursday8	9			
WaCharlie	Thursday9	9			
WaKilo	Sunday1	4			
WaKilo	Sunday2	4			
WaKilo	Sunday3	4			
WaKilo	Sunday7	2			
WaKilo	Sunday8	2			
WaKilo	Sunday9	4			
WaKilo	Monday7	4			
WaKilo	Monday8	9			
WaKilo	Monday9	9			
WaKilo	Tuesday7	9			
WaKilo	Tuesday8	9			
WaKilo	Tuesday9	-1			
WaKilo	Wednesday7	-1			
WaKilo	Wednesday8	-1			
WaKilo	Wednesday9	-1			
WaKilo	Thursday7	-1			
WaKilo	Thursday8	9			
WaKilo	Thursday9	-1			
WaLima	Sunday1	9			
WaLima	Sunday2	-1			
WaLima	Sunday3	2			
WaLima	Sunday7	2			
WaLima	Sunday8	2			
WaLima	Sunday9	4			
WaLima	Monday7	4			

WaData Table

WAS	SPECIALTIES	spec
WaGolf	span	0
WaGolf	fren	0
WaGolf	biol	1
WaFoxtrot	span	1
WaFoxtrot	fren	0
WaFoxtrot	biol	1
WaEcho	span	0
WaEcho	fren	1
WaEcho	biol	0
WaDelta	span	0
WaDelta	fren	1
WaDelta	biol	0
WaBravo	span	0
WaBravo	fren	0
WaBravo	biol	0
WaAlpha	span	0
WaAlpha	fren	0
WaAlpha	biol	1
WaHotel	span	0
WaHotel	fren	0
WaHotel	biol	0
WaJuliet	span	0
WaJuliet	fren	0
WaJuliet	biol	0
WaCharlie	span	0
WaCharlie	fren	0
WaCharlie	biol	1
WaKilo	span	0
WaKilo	fren	0
WaKilo	biol	1
WaLima	span	1
WaLima	fren	0
WaLima	biol	0

WaWorkload Table

WAS	workload
WaGolf	2
WaFoxtrot	2
WaEcho	4
WaDelta	5
WaBravo	1
WaAlpha	1
WaHotel	2
WaJuliet	2

WaCharlie	1
WaKilo	1
WaLima	2

ShiftData Table

SHIFTS	target
Sunday1	1
Sunday2	1
Sunday3	1
Sunday7	2
Sunday8	2
Sunday9	1
Monday7	2
Monday8	2
Monday9	1
Tuesday7	2
Tuesday8	1
Tuesday9	1
Wednesday7	2
Wednesday8	1
Wednesday9	1
Thursday7	2
Thursday8	1
Thursday9	1

Appendix C – AMPL .run Code, .dat Code, and Solution

Run File

```
# File final.run
model final.mod;
data final.dat;

option solver cplex;
solve;
display work;

table Results OUT "ODBC" "output.xls":
[WAS,SHIFTS],work;

write table Results;
```

Data File

```
# AMPL File final.dat

set SPECIALTIES := span,fren,biol;

table WaWorkload IN "ODBC" "input.xls":
WAS <- [WAS], workload;

table ShiftData IN "ODBC" "input.xls":
SHIFTS <- [SHIFTS], target;

table WaData IN "ODBC" "input.xls":
[WAS,SPECIALTIES], spec;

table Preferences IN "ODBC" "input.xls":
[WAS, SHIFTS], pref;

read table WaWorkload;
read table WaData;
read table ShiftData;
read table Preferences;

binary
piecewise
card
```

Solution

```
AMPL final.run
```

```

CPLEX 9.1.3: ILOG CPLEX, licensed to "AMPL Student
Edition for Taha Textbook".
optimal integer solution within mipgap or absmipgap;
objective 1607.222222
354 MIP simplex iterations
35 branch-and-bound nodes
absmipgap = 0.0987654, relmipgap = 6.1451e-05
work [*,*] (tr)
# $3 = WaCharlie
# $6 = WaFoxtrot
# $8 = WaHotel
# $9 = WaJuliet
:
      WaAlpha WaBravo '$3' WaDelta WaEcho '$6'
WaGolf '$8' '$9' WaKilo WaLima :=
Monday7      0      0      0      0      1      0      1
0 0      0      0
Monday8      0      0      0      0      0      0      0
0 1      0      0
Monday9      0      0      0      0      0      0      0
0 0      1      0
Sunday1      0      0      0      0      1      0      0
0 0      0      0
Sunday2      0      1      0      0      0      0      0
0 0      0      0
Sunday3      0      0      0      0      0      0      0
1 0      0      0
Sunday7      0      1      0      0      0      0      0
0 1      0      0
Sunday8      0      0      0      0      1      0      0
1 0      0      0
Sunday9      0      0      0      1      0      0      0
0 0      0      0
Thursday7    0      0      0      1      0      1      0
0 0      0      0
Thursday8    0      0      1      0      0      0      0
0 0      0      0
Thursday9    0      0      0      1      0      0      0
0 0      0      0
Tuesday7     0      0      0      0      0      1      0
0 0      0      0
Tuesday8     0      0      0      0      0      0      1
0 0      0      0
Tuesday9     0      0      0      0      0      0      0
0 0      0      1
Wednesday7   0      0      0      0      1      0      0
0 0      0      1
Wednesday8   0      0      0      1      0      0      0
0 0      0      0
Wednesday9   0      0      0      1      0      0      0
0 0      0      0
;

```

Appendix D – Test Output Data

WAS	SHIFTS	work	WaCharlie	Tuesday9	0	WaFoxtrot	Sunday9	0
WaAlpha	Sunday1	0	WaCharlie	Wednesday7	0	WaFoxtrot	Monday7	0
WaAlpha	Sunday2	1	WaCharlie	Wednesday8	0	WaFoxtrot	Monday8	0
WaAlpha	Sunday3	0	WaCharlie	Wednesday9	0	WaFoxtrot	Monday9	0
WaAlpha	Sunday7	0	WaCharlie	Thursday7	0	WaFoxtrot	Tuesday7	1
WaAlpha	Sunday8	0	WaCharlie	Thursday8	1	WaFoxtrot	Tuesday8	0
WaAlpha	Sunday9	0	WaCharlie	Thursday9	0	WaFoxtrot	Tuesday9	0
WaAlpha	Monday7	0	WaDelta	Sunday1	0	WaFoxtrot	Wednesday7	0
WaAlpha	Monday8	0	WaDelta	Sunday2	0	WaFoxtrot	Wednesday8	0
WaAlpha	Monday9	0	WaDelta	Sunday3	0	WaFoxtrot	Wednesday9	0
WaAlpha	Tuesday7	0	WaDelta	Sunday7	0	WaFoxtrot	Thursday7	1
WaAlpha	Tuesday8	0	WaDelta	Sunday8	0	WaFoxtrot	Thursday8	0
WaAlpha	Tuesday9	0	WaDelta	Sunday9	1	WaFoxtrot	Thursday9	0
WaAlpha	Wednesday7	0	WaDelta	Monday7	0	WaGolf	Sunday1	0
WaAlpha	Wednesday8	0	WaDelta	Monday8	0	WaGolf	Sunday2	0
WaAlpha	Wednesday9	0	WaDelta	Monday9	0	WaGolf	Sunday3	0
WaAlpha	Thursday7	0	WaDelta	Tuesday7	0	WaGolf	Sunday7	0
WaAlpha	Thursday8	0	WaDelta	Tuesday8	0	WaGolf	Sunday8	0
WaAlpha	Thursday9	0	WaDelta	Tuesday9	0	WaGolf	Sunday9	0
WaBravo	Sunday1	0	WaDelta	Wednesday7	0	WaGolf	Monday7	1
WaBravo	Sunday2	0	WaDelta	Wednesday8	1	WaGolf	Monday8	0
WaBravo	Sunday3	0	WaDelta	Wednesday9	1	WaGolf	Monday9	0
WaBravo	Sunday7	1	WaDelta	Thursday7	1	WaGolf	Tuesday7	0
WaBravo	Sunday8	0	WaDelta	Thursday8	0	WaGolf	Tuesday8	1
WaBravo	Sunday9	0	WaDelta	Thursday9	1	WaGolf	Tuesday9	0
WaBravo	Monday7	0	WaEcho	Sunday1	1	WaGolf	Wednesday7	0
WaBravo	Monday8	0	WaEcho	Sunday2	0	WaGolf	Wednesday8	0
WaBravo	Monday9	0	WaEcho	Sunday3	0	WaGolf	Wednesday9	0
WaBravo	Tuesday7	0	WaEcho	Sunday7	0	WaGolf	Thursday7	0
WaBravo	Tuesday8	0	WaEcho	Sunday8	1	WaGolf	Thursday8	0
WaBravo	Tuesday9	0	WaEcho	Sunday9	0	WaGolf	Thursday9	0
WaBravo	Wednesday7	0	WaEcho	Monday7	1	WaHotel	Sunday1	0
WaBravo	Wednesday8	0	WaEcho	Monday8	0	WaHotel	Sunday2	0
WaBravo	Wednesday9	0	WaEcho	Monday9	0	WaHotel	Sunday3	1
WaBravo	Thursday7	0	WaEcho	Tuesday7	0	WaHotel	Sunday7	0
WaBravo	Thursday8	0	WaEcho	Tuesday8	0	WaHotel	Sunday8	1
WaBravo	Thursday9	0	WaEcho	Tuesday9	0	WaHotel	Sunday9	0
WaCharlie	Sunday1	0	WaEcho	Wednesday7	1	WaHotel	Monday7	0
WaCharlie	Sunday2	0	WaEcho	Wednesday8	0	WaHotel	Monday8	0
WaCharlie	Sunday3	0	WaEcho	Wednesday9	0	WaHotel	Monday9	0
WaCharlie	Sunday7	0	WaEcho	Thursday7	0	WaHotel	Tuesday7	0
WaCharlie	Sunday8	0	WaEcho	Thursday8	0	WaHotel	Tuesday8	0
WaCharlie	Sunday9	0	WaEcho	Thursday9	0	WaHotel	Tuesday9	0
WaCharlie	Monday7	0	WaFoxtrot	Sunday1	0	WaHotel	Wednesday7	0
WaCharlie	Monday8	0	WaFoxtrot	Sunday2	0	WaHotel	Wednesday8	0
WaCharlie	Monday9	0	WaFoxtrot	Sunday3	0	WaHotel	Wednesday9	0
WaCharlie	Tuesday7	0	WaFoxtrot	Sunday7	0	WaHotel	Thursday7	0
WaCharlie	Tuesday8	0	WaFoxtrot	Sunday8	0	WaHotel	Thursday8	0

WaHotel	Thursday9	0	WaKilo	Sunday1	0	WaLima	Sunday2	0
WaJuliet	Sunday1	0	WaKilo	Sunday2	0	WaLima	Sunday3	0
WaJuliet	Sunday2	0	WaKilo	Sunday3	0	WaLima	Sunday7	0
WaJuliet	Sunday3	0	WaKilo	Sunday7	0	WaLima	Sunday8	0
WaJuliet	Sunday7	1	WaKilo	Sunday8	0	WaLima	Sunday9	0
WaJuliet	Sunday8	0	WaKilo	Sunday9	0	WaLima	Monday7	0
WaJuliet	Sunday9	0	WaKilo	Monday7	0	WaLima	Monday8	0
WaJuliet	Monday7	0	WaKilo	Monday8	0	WaLima	Monday9	0
WaJuliet	Monday8	1	WaKilo	Monday9	1	WaLima	Tuesday7	0
WaJuliet	Monday9	0	WaKilo	Tuesday7	0	WaLima	Tuesday8	0
WaJuliet	Tuesday7	0	WaKilo	Tuesday8	0	WaLima	Tuesday9	1
WaJuliet	Tuesday8	0	WaKilo	Tuesday9	0	WaLima	Wednesday7	1
WaJuliet	Tuesday9	0	WaKilo	Wednesday7	0	WaLima	Wednesday8	0
WaJuliet	Wednesday7	0	WaKilo	Wednesday8	0	WaLima	Wednesday9	0
WaJuliet	Wednesday8	0	WaKilo	Wednesday9	0	WaLima	Thursday7	0
WaJuliet	Wednesday9	0	WaKilo	Thursday7	0	WaLima	Thursday8	0
WaJuliet	Thursday7	0	WaKilo	Thursday8	0	WaLima	Thursday9	0
WaJuliet	Thursday8	0	WaKilo	Thursday9	0			
WaJuliet	Thursday9	0	WaLima	Sunday1	0			

Appendix E – Pivot Table

A value of 1 in row i, column j indicates that in the optimal solution, WA i works shift j.

WAS	Sunday1	Sunday2	Sunday3	Sunday7	Sunday8	Sunday9	Monday7	Monday8
WaAlpha		1						
WaBravo				1				
WaCharlie								
WaDelta						1		
WaEcho	1				1		1	
WaFoxtrot								
WaGolf							1	
WaHotel			1		1			
WaJuliet				1				1
WaKilo								
WaLima								
(blank)								

Monday9	Tuesday7	Tuesday8	Tuesday9	Wednesday7	Wednesday8	Wednesday9	Thursday7	Thursday8
								1
					1	1	1	
	1						1	
		1						
1				1	1			
Thursday9								
	1							